

On the Inference of Michelson-Morley Experiment: Fitzgerald Contraction Implication on the Time Dimension

Radwan M. Kassir¹

Abstract

The implication of the Fitzgerald length contraction hypothesis on the time dimension is considered. Originally set as an ad hoc interpretation of the Michelson-Morley experiment null result, the hypothesis is expressed in terms of a space transformation equation inferred from the Galilean transformation, leading to a time conversion exhibiting a contractive property.

Keywords: Fitzgerald contraction; time dilation; Michelson-Morley experiment; space transformation

1. Introduction

The Michelson-Morley experiment was designed in the late 19th century to detect the ether (a conjectured light propagation medium) 'wind' created by the earth motion through the ether-filled space.

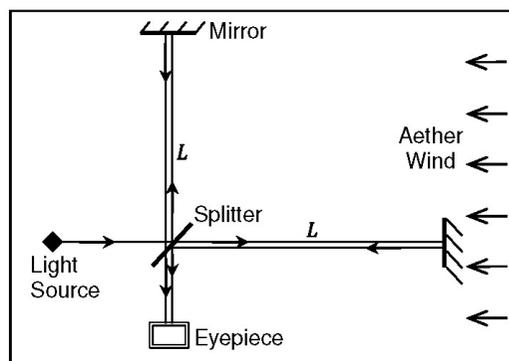


Fig.1 Michelson-Morley experiment setting

¹ D.A.H. (S & P), Mechanical Dept., Verdun St., PO Box 11-7159, Beirut 1107 2230, Lebanon

As light was supposed to travel at a constant speed with respect to the ether, the relative speed of light with respect to earth would then depend on the light propagation direction with respect to the ether 'wind' direction. Fig.1 illustrates the experiment principle. A light beam is sent to a semi silvered mirror placed at 45° angle to the beam direction, splitting it into two beams with directions perpendicular to each other. Each of the two split beams will then travel a distance L from the splitter before it is reflected back to it, and recombining with the other reflected beam in an eyepiece, producing an interference pattern. If the earth is moving through the ether, it would create an ether 'wind' blowing in the opposite direction to its motion, thus delaying the back-and-forth trip of the beam traveling longitudinally to the ether 'wind', with respect to the beam with the transverse motion. This time delay will cause the recombined beams to be out phase, thus a shift in the fringes from the position that would be expected under symmetrical beam trips was anticipated. However, no such shift was observed, even with much more sophisticated variations of the experimental setting providing very high accuracy of the measurements.

If the speed of light with respect to the ether is given as C , and the earth relative velocity as V , then it can be shown that the total longitudinal travel time can be expressed as (derivation will be subsequently presented):

$$T_l = \frac{2L}{C} \frac{1}{1 - \frac{V^2}{C^2}}.$$

Whereas, the total travel time for the transverse beam was $T_t = 2L/C$, as originally indicated by Michelson. Thus T_l is greater than T_t by a factor of $1/(1 - V^2/C^2)$. However, this factor was reduced as a corrected transverse travel time of

$$T_t = \frac{2L}{C} \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}}$$

was introduced by Lorentz, taking into consideration the light beam drifting velocity. However, this new time difference still couldn't be reconciled, as the experiment exhibited null result in terms of fringe shift.

In an attempt to resolve this discrepancy, a length contraction hypothesis was proposed by Fitzgeraldⁱⁱ and Lorentzⁱⁱⁱ. According to this hypothesis, an object would contract along the direction of its motion by a factor of $1/\gamma$, with

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}}$$

being the Lorentz factor. It follows that, the light beam will end up traveling back and forth the contracted longitudinal distance L/γ , and the longitudinal travel time becomes:

$$T_l = \frac{2L/\gamma}{C} \frac{1}{1 - \frac{V^2}{C^2}} = \frac{2L}{C} \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}}$$

which is the same as the transverse travel time, thus justifying the Michelson-Morley null result.

In this paper, the implication of the Fitzgerald contraction hypothesis on the time dimension is considered.

2. Lorentz Factor – Physical Perspective

Starting back from the ether theory and the Michelson-Morley experiment null result, the Lorentz factor is considered in the context of a physical overview, prior to attempting a mathematical reconciliation formulation.

In a certain setting (Fig. 2a), where the ether is assumed to be totally dragged by the earth, a light beam, having a velocity C with respect to the ether, is to travel a total round trip distance of $2L$ (L being the length of each of the two orthogonal arms of the utilized apparatus), with respect to the earth. Let K_o be a stationary frame of reference with respect to the ether, and K'_o be the earth reference frame; there is no relative motion between K_o and K'_o . The time it takes the light beam to complete the round trip, in either longitudinal or transversal arm direction, as measured by an observer in K_o , or K'_o , will be

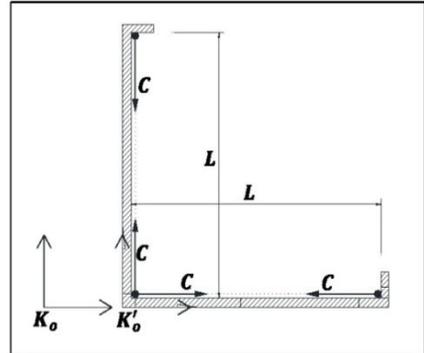


Fig.2a Velocity diagram w.r.t. K_o : Ether is assumed to be totally dragged—No relative motion between ether and earth

$$T_o = \frac{2L}{C} . (1)$$

In a different setting (Fig. 2b), the earth is moving through the ether at a constant speed V . An ether 'wind' of speed V will be thus created with respect to the earth. Two light beams are considered. One beam is to travel a total round trip ground distance of $2L$, going back and forth along the direction of the earth motion. A similar round trip in the transverse direction is to be travelled by the other beam. Let $K(X,Y,Z)$ be a frame of reference at rest with respect to the ether, and $K'(X',Y',Z')$ be a reference frame attached to earth; K and K' are in relative motion with velocity V .

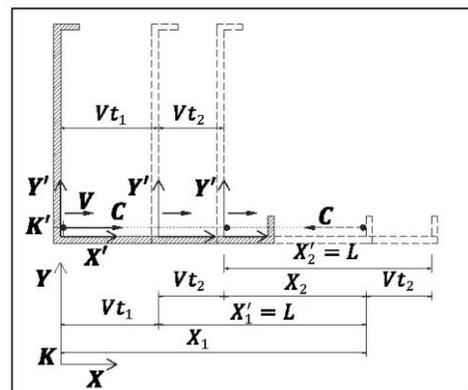


Fig. 2b Longitudinal velocity diagram w.r.t. K : Earth is traveling at velocity V w.r.t. the ether— K' is in relative motion w.r.t. K .

2.1. Longitudinal Travel Time

According to the ether theory, the velocity of the light beam with respect to K (the ether frame) is equal to C . Referring to Fig.2b, we can write

$$X_1 = Vt_1 + X'_1, (2)$$

and

$$X_2 = -Vt_2 + X'_2, (3)$$

where t_1 and t_2 are the forward and backward longitudinal travel time, respectively. Substituting $X_1 = Ct_1$, and $X_2 = Ct_2$, in (2) and (3), and solving for t_1 and t_2 , the total round trip time T_l will be determined as

$$T_l = \frac{L}{C-V} + \frac{L}{C+V} ,$$

$$T_l = \frac{2L/C}{1 - \frac{V^2}{C^2}} . (4)$$

For a stationary observer in K' , the ether is 'running' at velocity V in the longitudinal direction, and the light beam upstream and downstream velocities are $C - V$ and $C + V$, respectively, according to the Galilean velocity transformation. Hence, the longitudinal travel time with respect to an observer in K' will be also given by (4).

2.2. Transverse Travel Time

For a stationary observer in K' , the ether is relatively 'flowing' at speed V in the longitudinal direction. The transverse light beam is traveling in the Y' direction with respect to K' , at velocity \bar{C} with respect to K . Using the Galilean velocity transformation, the light beam relative velocity \bar{C}' with respect to K' can be expressed by the following vector addition (Fig 2c).

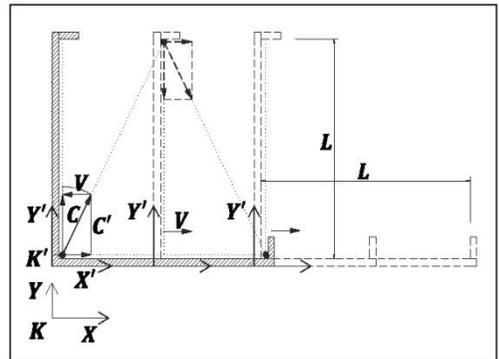


Fig. 2c Transverse velocity diagram w.r.t. K : Earth is traveling at velocity V w.r.t. the ether— K' is in relative motion w.r.t K .

$$\bar{C}' = \bar{C} - \vec{V} . (5)$$

Therefore, $C' = \sqrt{C^2 - V^2}$, or

$$C' = C \sqrt{1 - \frac{V^2}{C^2}} \quad . (6)$$

It follows that the transverse round trip travel time can be expressed as,

$$T_t = \frac{2L}{C'} = \frac{2L/C}{\sqrt{1 - \frac{V^2}{C^2}}} \quad . (7)$$

Alternatively, with respect to K , the light beam one way transverse distance Ct_1 can be expressed as $C^2t_1^2 = L^2 + V^2t_1^2$, yielding $t_1 = L / \sqrt{C^2 - V^2}$. Therefore,

$$T_t = \frac{2L/C}{\sqrt{1 - \frac{V^2}{C^2}}}$$

returning equation (7).

In either approach, the resulting travel time is expressed as the ratio of the arm length to the relative velocity of the light beam with respect to K' .

2.3. Length Contraction Hypothesis

In order to validate the ether 'wind' conjecture, following Michelson-Morley null result, the longitudinal and the transverse travel time, T_l and T_t , must be equal. This could be made possible if a space-time modifying transformation was assumed. In fact, comparing (4) and (5), the longitudinal travel time is scaled with respect to the transverse time by a factor of

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}} \quad . (8)$$

It is then postulated that the length of a moving object would be contracted along its motion direction by a factor of $1/\gamma$. When this principle is applied in our case to the moving arm of length L , traveling with respect to K at speed V , the longitudinal travel time of the light beam would become just equal to the transverse time. In fact, equation (4) becomes,

$$T_l = \frac{\frac{2L}{C} \frac{1}{\gamma}}{\left(1 - \frac{V^2}{C^2}\right)} = \frac{\frac{2L}{C} \frac{1}{\gamma}}{\frac{1}{\gamma^2}} = \frac{2L}{C} \gamma = \frac{2L/C}{\sqrt{1 - \frac{V^2}{C^2}}} . \quad (9)$$

It follows from (1), (7), and (9) that,

$$T_l = T_t = \gamma T_o , \quad (10)$$

and

$$L_c = \frac{L}{\gamma} . \quad (11)$$

Where L_c is the contracted length with respect to K , and γ is the Lorentz factor.

Equations (10) and (11) can be interpreted as the time dimension is dilated, and the length's is contracted with respect to the ether frame, due to the earth relative motion, with γ being the time dilation, and $1/\gamma$ the length contraction factors. Yet, in actuality, the Fitzgerald contraction causes the longitudinal travel time to contract from $\gamma^2(2L/C)$ to $\gamma(2L/C)$ —but still dilated by a factor of γ with respect to $T_o = 2L/C$. Whether this is a valid interpretation of an actual time dilation will be evaluated in the mathematical perspective analysis in Section 4.

3. Special Relativity's Interpretation

In special relativity, the ether conjecture was abandoned, and replaced by the principle of the constancy of the speed of light in all inertial frames of reference, as postulated by Einstein^{iv}. In contrast with the ether theory, this principle is in fact comparable to viewing the "ether" as if it were at rest with respect to any inertial frame of reference, which makes the speed of light constant (equal to C) with respect to any corresponding observer.

It follows that, the particular studied case of the relative motion of the ether-earth reference frames can be generalized to any pair of reference frames in relative motion with any relative velocity less than C , with the only difference being the rest state of the "ether" with respect to an observer in K' , which brings the speed of light in K' to C . Hence, the reference frame K' , which is in relative motion with respect to the 'stationary' frame K , becomes equivalent to K_o , where the light round trip travel time is $2L/C$ for both the longitudinal and transverse directions.

Therefore, as a consequence of the special relativity postulate, equations (10) and (11) reduce to

$$T = \gamma T' = \frac{2L'/C}{\sqrt{1 - \frac{V^2}{C^2}}}, \quad (12)$$

and

$$L = \frac{L'}{\gamma} = L' \sqrt{1 - \frac{V^2}{C^2}}. \quad (13)$$

Where T and T' are the travel time, L and L' the arm length, with respect to K and K' , respectively.

Now, (12) can be written as,

$$\Delta t = \gamma \Delta t',$$

or

$$t - t_o = \gamma (t' - t'_o). \quad (14)$$

Where t_o is a reference time point on the t -axis in K , and t'_o is the corresponding time coordinate in K' . If t_o and t'_o were chosen to be the time coordinates of the origins of K and K' , respectively, they can be set to zero, had we assumed that at $t_o = 0$ and $t'_o = 0$, K and K' are coinciding. It follows from (14) that, from the perspective of the frame origins, the K time coordinate with respect to that of K' can be stated as,

$$t = \gamma t'. \quad (15)$$

4. Mathematical Perspective

Since the obtained travel time in the reference frames K and K' seems to involve time transformation, the time coordinate should be introduced to the reference frames. Thus, K and K' are now represented as $K(X, Y, Z, t)$ and $K'(X', Y', Z', t')$.

With respect to an observer in K , the hypothesized Fitzgerald length contraction can be expressed by the equation

$$X = Vt + \frac{X'}{\gamma}, \quad (16)$$

inferred from the Galilean transformation. Equation (16) can be rearranged to the following transformation expression.

$$X' = \gamma (X - Vt). \quad (17)$$

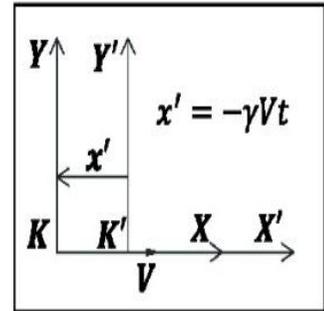


Fig. 3a X' -coordinate (x') of the origin of K .

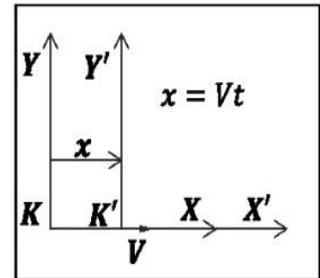


Fig. 3b X -coordinate (x) of the origin of K' .

Applying the space transformation given by equation (17), the X -coordinate of the origin of K ($X = 0$) has a transformed X' -coordinate of

$$x' = -\gamma Vt \quad (18)$$

at time t (Fig. 3a). Similarly, applying the same transformation equation (17), the X -coordinate of the origin of K'

$$x = Vt \quad (19)$$

has a transformed X' -coordinate of $X' = 0$ (the origin of K') at the same instant of time t (Fig. 3b), with respect to K ; i.e., for $\Delta t = 0$ ($x = Vt$ and $x' = -\gamma Vt$ are simultaneous events with respect to K), the distance between the frame origins is expressed by equations (18) and (19) from the perspective of K .

It follows from equations (18) and (19) that

$$x = -\frac{x'}{\gamma},$$

which indicates a 'distance' contraction with respect to K (i.e., with respect to an observer in K , the travelled distance x by the K' origin at a certain time instant t , is contracted relative to the absolute value of the corresponding K origin coordinate x' relative to K' , attained at the same instant of time t). However, this distance scaling is not in line with the Fitzgerald hypothesis physical interpretation (i.e., the contraction of a length interval relatively moving in the direction of the relative motion).

On the other hand, since K is traveling at velocity $-V$ relative to K' , then the distance x' (corresponding to $X = 0$) must be equal to $-Vt'$. Substituting x' in equation (18), valid for $X = 0$, we get

$$-Vt' = -\gamma Vt,$$

yielding

$$t = \frac{t'}{\gamma},$$

which is a time contraction with respect to K for $X = 0$ (i.e., with respect to an observer at the K origin, the time it takes K' origin to travel a certain distance x with respect to K , corresponding to the absolute value of the attained K origin coordinate x' with respect to K' , is contracted relative to the corresponding time t' in K').

It follows that the Fitzgerald contraction, expressed by equation (17), mathematically results in a time contraction with respect to K , which is not in line with the physically derived equations (10) and (15), interpreted as exhibiting a time dilation with respect to K , for the hypothesized length contraction.

5. Conclusion

For two reference frames relatively moving at a uniform velocity, it is shown that the Fitzgerald contraction hypothesis can be physically interpreted to imply dilation of the time dimension with respect to the stationary frame. Whereas, the hypothesis mathematical formulation results in a space contraction transformation exhibiting time contraction with respect to the stationary frame. Hence, the physically anticipated time dilation of the Fitzgerald contraction is not mathematically reconciled.

ⁱⁱFitzGerald, G.F., *The Ether and the Earth's Atmosphere*. Science 1889. **328** (13): p. 390.

ⁱⁱⁱLorentz, H.A., *The Relative Motion of the Earth and the Aether*. Zittingsverlag Akad. V. Wet. , 1892. **1** p. 74–79

^{iv}Einstein, A., *Zur Elektrodynamik bewegter Körper*. Annalen der Physik 1905. **322**(10): p. 891–921.